1032 機械系博士班資格考試題目

考試科目	方式	
工程數學	Closed Book, 不可使用計算機, 共 9 題採計 6 題	Part I

1. **Using the method of variation of parameters to solve the differential equation (17%)**

$$y'' + y = \sec x$$

2. **Find a solution of the following equation (17%)**

$$y'' - 4y' + 4y = 0 \quad \text{with} \quad y(0) = 3 \quad , \quad \left.\frac{dy}{dx}\right|_{x=0} = 1$$

3. **Using the method of Laplace Transformation to solve the initial value problem of $y(t)$ (17%)**

$$y'' - y = t \quad \text{with} \quad y(0) = 1 \quad , \quad \left.\frac{dy}{dt}\right|_{t=0} = 1$$

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1. Find det(B) and det(C), with the given det(A) = 5. (17%)

$$A=\begin{pmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{pmatrix},\quad B=\begin{pmatrix} 2a_1 & a_2 & a_3 \\ 6b_1 & 3b_2 & 3b_3 \\ 2c_1 & c_2 & c_3 \end{pmatrix},\quad C=\begin{pmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{pmatrix}$$

2. If a matrix A has been diagonalized as D = $\begin{pmatrix} 4 & 0 \\ 0 & 6 \end{pmatrix}$, and its eigenvectors are $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$, find the matrix A by showing detail processes. (17%)

3. Find the length of the curve of circular helix $\vec{r}(t) = 2\cos t\,\vec{i} + 2\sin t\,\vec{j} + 6t\,\vec{k}$ from (2, 0, 0) to (2, 0, 24 π). (17%)

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1. (17%) Derive the d'Alembert's solution of the following partial differential equation.

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \quad (-\infty < x < \infty, \ 0 < t < \infty);$$
$$u(x,0) = f(x), \ u_t(x,0) = g(x) \quad (-\infty < x < \infty).$$

2. (17%) Consider a laterally insulated bar of length L whose ends are kept at temperature 0 and 100, respectively, assuming that the initial temperature is

$$f(x) = \begin{cases} x, & 0 < x < L/2 \\ L - x, & L/2 < x < L \end{cases}$$

(a) Write down the required governing equation and the corresponding boundary and initial conditions.

(b) Find the temperature of the bar with the method of separation of variables.

3. (17%) Represent the following function *f(x)* by (a) a Fourier series, (b) a Fourier cosine series, and (c) a Fourier sine series. Graph the corresponding periodic extensions of *f(x)*, respectively?

$$f(x) = \begin{cases} 3 & 0 < x < 1 \\ -1 & 1 < x < 2 \end{cases}$$