1. Solve the following differential equations: (17%)

    (a) $y'' + 6y' + 9y = 50e^{-x}\cos x$ (8%)

    (b) Choose a constant α so that the differential equation is exact, and then obtain the general solution (9%).

    $2xy^3 - 3y - \left(3x + \alpha x^2 y^2 - 2\alpha y\right)y' = 0$

2. Solve $y'' + 9y = 9\csc 3x$ by using variation of parameters method (Wronskian) (17%)

3. Solve the following system of differential equations. (17%)

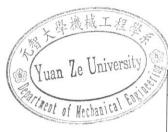 $$y' = \begin{bmatrix} 3 & -3 \\ 3 & 3 \end{bmatrix} y$$

1. Evaluate the integral $I = \int_C [(y^2 - 6xy + 6)dx + (2xy - 3x^2)dy]$ if C has the initial point A: (-1, 0) and terminal point B: (3, 4). **(17%)**

2. Find the inverse $\mathbf{A}^{-1}$ of **(17%)**

$$\mathbf{A} = \begin{bmatrix} 8 & 0 & 1 \\ 3 & -2 & 1 \\ 1 & 4 & 0 \end{bmatrix}$$

3. Find the eigenvalues and eigenvectors of the matrix **(17%)**

$$\mathbf{A} = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$$

1. (17%) Find the two basic half-range expansions (even and odd) of the following function. Sketch $f(x)$ and its two periodic extensions.

$$f(x) = \begin{cases} 1-x, & 0 < x < 1 \\ x-1, & 1 < x < 2 \end{cases}$$

2. (17%) Solve the following partial differential equation for $u(x, t)$ first and then plot the distribution of $u(x, t)$ vs. x at different t's.

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad (0 < x < L, \; 0 < t < \infty);$$
$$u(0,t) = 0, \; u(L,t) = 0, \quad (0 < t < \infty).$$
$$u(x,0) = x$$

3. (17%) Solve the following Laplace equation with the given boundary conditions.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad (0 < x < a, \; 0 < y < b);$$
$$u(x,0) = 0, \quad u(x,b) = 0 \quad (0 < x < a);$$
$$u(0,y) = 0, \quad u(a,y) = 100, \quad (0 < y < b).$$