1052 機械系博士班資格考試題目

考試科目	方式	
工程數學	**Closed Book, 不可使用計算機, 共 9 題採計 6 題**	**Part I**

1. Solve the following problems of the first order differential equations: (17%)
   (a) Explain what is the meaning of an integration factor? For the following typical linear differential equation, what is its integration factor? What is the solution? (10%)
   $$\frac{dy}{dx} + p(x)y = q(x)$$
   (b) Choose a constant α so that the differential equation is exact, and then obtain the general solution (7%)
   $$2xy^3 - 3y - (3x + \alpha x^2 y^2 - 2\alpha y)y' = 0$$

2. Solve the following second order differential equations: (17%)
   (a) $y'' + 4y = (x^2 - 3)\sin 2x$ (8%)
   (b) Find the general solution of the Euler-Cauchy equation. (9%).
   $$x^2 y'' - 5xy' + 9y = 0$$

3. Solve the following problems relating to Laplace Transform. (17%)
   (a) System of differential equaitons: $\begin{cases} y_1'' = y_1 + 3y_2 \\ y_2'' = 4y_1 + 2y_2 \end{cases}$ (9%)
   (b) Solve the following differential equation, where *u(t-2)* is a unit step function: (8%)
   $$y'' + y = u(t-2),\ y(0) = 0,\ y'(0) = 0$$

For your reference:

f(t)	1	t	e^{at}	te^{at}	$\cos(\omega t)$	$\sin(\omega t)$	u(t-a)	$\delta(t-a)$
L (f)	1/s	$1/s^2$	1/(s-a)	$1/(s-a)^2$	$s/(s^2+\omega^2)$	$\omega/(s^2+\omega^2)$	e^{-as}/s	e^{-as}

$L(f') = s\,L(f) - f(0)$ $L(f'') = s^2\,L(f) - sf(0) - f'(0)$, $L[f(t-a)u(t-a)] = e^{-as}F(s)$

$L[e^{at}f(t)] = F(s-a)$, $L\left[\int_0^t f(\tau)d\tau\right] = \frac{1}{s}F(s)$

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1. Evaluate the integral $I = \int_c [(y^2 - 6xy + 6)dx + (2xy - 3x^2)dy]$ if C has the initial point *A*: (-1, 0) and terminal point *B*: (3, 4). (Hint: By the Potential Theorem) **(17%)**

2. Find the inverse $\mathbf{A}^{-1}$ of **(17%)**

$$\mathbf{A} = \begin{bmatrix} 2 & 0 & 1 \\ -2 & 3 & 4 \\ -5 & 5 & 6 \end{bmatrix}$$

3. Find the eigenvalues and eigenvectors of the matrix **(17%)**

$$\mathbf{A} = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$$

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1. (17%) Find the two basic half-range expansions (even and odd) of the following function. Sketch $f(x)$ and its two periodic extensions.

$$f(x) = \begin{cases} \dfrac{2x}{L}, & 0 < x < \dfrac{L}{2} \\ \dfrac{2(L-x)}{L}, & \dfrac{L}{2} < x < L \end{cases}$$

2. (17%) Solve the following partial differential equation for $u(x, t)$ first and then plot the distribution of $u(x, t)$ vs. x at different t's.

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, \quad (0 < x < L,\ 0 < t < \infty)$$

$$u(0,t) = 0,\ u(L,t) = 0, \quad \text{for all } t \geq 0$$

$$u(x,0) = f(x),\ u(x,0) = 0 \quad \text{(where } f(x) \text{ is the same as problem \#1 above)}$$

3. (17%) Solve the following partial differential equation for $u(x, t)$ first and then plot the distribution of $u(x, t)$ vs. x at different t's.

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad (0 < x < L,\ 0 < t < \infty)$$

$$u(0,t) = 0,\ u(L,t) = 0, \quad (0 < t < \infty)$$

$$u(x,0) = \begin{cases} x, & 0 < x < \dfrac{L}{2} \\ L - x, & \dfrac{L}{2} < x < L \end{cases}$$