

1051 機械系博士班資格考試題目

考試科目	方式	
工程數學	Closed Book, 不可使用計算機, 共 9 題採計 6 題	Part I

1. Solve the following first order differential equations: (17%)

(a) Check for the exactness, if not, find an integrating factor and solve it

$$(\sin y \cos y + x \cos^2 y)dx + xdy = 0 \quad (9\%)$$

(b) Find the general solution of the differential equation. (8%).

$$y' + ky = e^{2kx}$$

2. Solve the following second order differential equations: (17%)

(a) $y'' + 6y' + 9y = e^{-3x} - 27x^3$ (8%)

(b) Find the general solution of the Euler-Cauchy equation. (9%).

$$x^2 y'' - xy' + 2y = 0$$

3. Solve the following system of differential equations. (17%)

(a)
$$\begin{cases} y_1' = 4y_1 + y_2 \\ y_2' = -y_1 + 2y_2 \end{cases} \quad (9\%)$$

(b) Solve the following differential equation by Laplace transform, where $u(t-1)$ is a unit step function: (8%)

$$y'' + y = u(t-1), \quad y(0) = -1, \quad y'(0) = 20$$

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1. Evaluate the integral $I = \int_C (3x^2 dx + 2yz dy + y^2 dz)$ if C has the initial point $A: (0, 1, 2)$ and terminal point $B: (1, -1, 7)$. (Hint: By the Potential Theorem) (17%)

2. Find the inverse \mathbf{A}^{-1} of (17%)

$$\mathbf{A} = \begin{bmatrix} 2 & 0 & 1 \\ -2 & 3 & 4 \\ -5 & 5 & 6 \end{bmatrix}$$

3. Find the eigenvalues and eigenvectors of the matrix (17%)

$$\mathbf{A} = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

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1. (a) Find the Fourier series of the following function $f(x)$ with period 4; (9%)
- (b) Find the Fourier integral representation of the function $g(x)$. (8%)

$$(a) f(x) = \begin{cases} -k, & -2 < x < 0 \\ k, & 0 < x < 2 \end{cases}; \quad (b) g(x) = \begin{cases} k, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$$

2. (17%) Solve the following partial differential equation for $u(x, t)$ first and then plot the distribution of $u(x, t)$ vs. x at different t 's.

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \quad (-\infty < x < \infty, \quad 0 < t < \infty);$$

$$u(x, 0) = \begin{cases} 2, & |x| < 1 \\ 0, & |x| > 1 \end{cases}; \quad u_t(x, 0) = 0 \quad (-\infty < x < \infty).$$

3. (17%) Solve the following Laplace equation with the given boundary conditions.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad (0 < x < a, \quad 0 < y < b);$$

$$u(x, 0) = 0, \quad u(x, b) = 0 \quad (0 < x < a);$$

$$\frac{\partial u}{\partial x}(a, y) = 0, \quad u(a, y) = 1, \quad (0 < y < b).$$