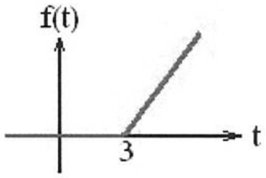
1071 機械系博士班資格考試題目

考試科目	方式	
Engineering Mathematics	**Closed Book, calculator is not allowed. You need to answer 6 problems out of the 9 problems overall.**	**Part I**

1. Solve the following non-homogeneous differential equations: (17%)
   (a) $y'' + 6y' + 9y = 50e^{-x}\cos x$ (8%)
   (b) $y'' + 4y = 2\sec x$ (9%)

2. Solve the following differential equations: (17%)
   (a) Find the solution for the following Bernoulli's equation. (5%)
   $$\frac{dy}{dx} + y = y^2,\ y(0) = -1$$
   (b) For the following differential equation, Find an integrating factor,(6%) and solve it (6%).
   $$(x^4 + y^2)\,dx - xy\,dy = 0$$

3. Solve $y'' + 4y = f(t)$ ， $y(0) = y'(0) = 0$ (17%)



For your reference:

f(t)	1	t	e^{at}	te^{at}	cos(ωt)	sin(ωt)	u(t-a)	$\delta(t-a)$
L (f)	1/s	$1/s^2$	1/(s-a)	$1/(s-a)^2$	$s/(s^2+\omega^2)$	$\omega/(s^2+\omega^2)$	e^{-as}/s	e^{-as}

$L(f') = s\,L(f) - f(0)$ $\quad L(f'') = s^2\,L(f) - sf(0) - f'(0),$ $\quad L[f(t-a)u(t-a)] = e^{-as}F(s)$

$L[e^{at}f(t)] = F(s-a),$ $\quad L\left[\int_0^t f(\tau)d\tau\right] = \frac{1}{s}F(s)$

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1. Evaluate the integral $I = \int_C (3x^2 dx + 2\, y\, z\, dy + y^2 dz)$ if C has the initial point *A*: (0, 1, 2) and terminal point *B*: (1, -1, 7). (Hint: By the Potential Theorem) **(17%)**

2. Find the inverse $\mathbf{A}^{-1}$ of **(17%)**

$$\mathbf{A} = \begin{bmatrix} 8 & 0 & 1 \\ 3 & -2 & 1 \\ 1 & 4 & 0 \end{bmatrix}$$

3. Find the eigenvalues and eigenvectors of the matrix **(17%)**

$$\mathbf{A} = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$$

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Engineering Mathematics	**Closed Book, calculator is not allowed. You need to answer 6 problems out of the 9 problems overall.**	**Part III**

1. (17%) (a) Find the two basic half-range expansions (even and odd) of the following function ($0 < x < 2$). (b) Sketch *f(x)* and its two periodic extensions. (c) What are the values of the two Fourier sine and cosine series at $x = 2$, respectively.

$$f(x) = \begin{cases} 3, & 0 < x < 1 \\ 0, & 1 < x < 2 \end{cases}$$

2. (17%) Solve the following partial differential equation for $u(x, y)$.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad (0 < x < a, \ 0 < y < b);$$

$$u(x,0) = 0, \quad u(x,b) = 100 \quad (0 < x < a);$$

$$u(0,y) = 0, \quad u(a,y) = 0, \quad (0 < y < b).$$

3. (17%) Solve the following partial differential equation for $u(x, t)$ first and then plot the distribution of $u(x, t)$ vs. x at different t's.

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad (0 < x < 2, \ 0 < t < \infty)$$

$$u(0,t) = 0, \quad u(2,t) = 0, \quad (0 < t < \infty)$$

$$u(x,0) = \sin(\pi x / 2) \quad (0 < x < 2)$$