Using the method of variation of parameters to solve the differential equation (17%)

$$y'' + y = \sec x$$

Find a solution of the following equation (17%)

$$y'' + 4y = x + 2e^{-2x}$$

Using the method of Laplace Transformation to solve the initial value problem of $y(t)$ (17%)

$$y'' + 4y' + 3y = e^t \quad \text{with} \quad y(0) = 0 \quad, \quad \left.\frac{dy}{dt}\right|_{t=0} = 2$$

一、若 $A = \begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$

    (a) 找出 A 之所有 eigenvalues 及 eigenvectors。(6%)

    (b) 將 A 對角線化 (diagonalization)並驗證之。(5%)

    (c) 若 $B = A^{50} - 4A^{20} + 5I$, 則 B 之 eigenvalues and eigenvectors 為何？(6%)

二、$f(x, y, z) = x^2 + y^2 - z^2$

    (a)則 grad(f) = ? (5%)

    (b)若 P 點位置為 P(1, 0, -1)，則在 P 點處往哪個方向的 f 值變化率最大？其值（大小）為何？(6%)

    (c)若 $\vec{v} = 2\vec{i} - \vec{j} + 5\vec{k}$, 則在 P 點往 $\vec{v}$ 方向的 $\dfrac{df}{ds} = ?$（或寫成 $D_{\vec{v}}f = ?$）(6%)

三、(a)將下列 linear system 改寫成 AX = B 形式。並以 Gauss-Jordan 法解之。(6%)

        $x_1 + 3x_2 - 2x_3 = -7$

        $4x_1 + x_2 + 3x_3 = 5$

        $2x_1 - 5x_2 + 7x_3 = 19$

    (b)以 cofactor expansion 計算 det(A)。(5%)

    (c) rank(A) = ? rank(A|B) = ? (6%)

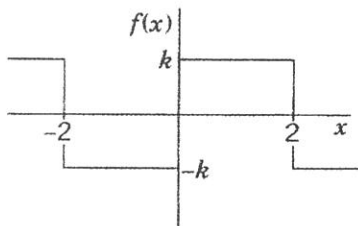| 考試科目	方式	
工程數學	Closed Book,不可使用計算機, 共 9 題採計 6 題	Part III

1. Solve $\dfrac{\partial u}{\partial x} = 4\dfrac{\partial u}{\partial y}$, $u(0, y) = 8e^{-3y}$ (17%)

2. Find Fourier series of the following function. (17%)

$$f(x) = \begin{cases} -k & if \ -2 < x < 0 \\ k & if \ \ \ 0 < x < 2 \end{cases} \quad p = 2L = -4, \ L = 2$$

3. For the following questions, try to explain and show that you fully understand their meanings and applications. (17%)

   (a) Fourier series. Why use Fourier series? (5%)

   (b) Complex Fourier series? Why use it? (5%)

   (c) What is Fourier integral? What is Fourier transforms? (7%)