991 機械系博士班資格考試題目

考試科目	方式	
工程數學	Closed Book,不可使用計算機, 共 9 題採計 6 題	Part I

1. Solve $yy' + xy^2 = x$ (17%)

2. For the following differential equation. (17%)
   $(x^2D^2 + 2xD - 12I)y = 1/x^3$

3. Find a general solution of : (17%)
   $\begin{cases} y_1' = y_1 + y_2 + \sin t \\ y_2' = 4y_1 + y_2 \end{cases}$

Note: Laplace Transform (For your reference only, if you prefer to use it.)

f(t)	1	t	e^{at}	te^{at}	$\sin(\omega t)$	u(t-a)
L (f)	1/s	$1/s^2$	1/(s-a)	$1/(s-a)^2$	$\omega/(s^2+\omega^2)$	e^{-as}/s

$L(f') = sL(f) - f(0)$

$L(f'') = s^2 L(f) - sf(0) - f'(0)$

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Evaluate the integral $I = \int_C [(y^2 - 6xy + 6)dx + (2xy - 3x^2)dy]$ if C has the initial point A: (-1, 0) and terminal point B: (3, 4). **(17%)**

Find the inverse $\mathbf{A}^{-1}$ of **(17%)**

$$\mathbf{A} = \begin{bmatrix} 8 & 0 & 1 \\ 3 & -2 & 1 \\ 1 & 4 & 0 \end{bmatrix}$$

Find the eigenvalues and eigenvectors of the matrix **(17%)**

$$\mathbf{A} = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$$

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1. (17%) (a)Find the two basic half-range expansions (even and odd) of the following function. Sketch $f(x)$ and its two periodic extensions. (b) What are the values of the two Fourier sine and cosine series at $x = 4$, respectively.

$$f(x) = \begin{cases} 2, & 0 < x < 1 \\ 0, & 1 < x < 2 \end{cases}$$

2. (17%) Solve the following Laplace equation with the given boundary conditions.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad (0 < x < 2, \; 0 < y < 1);$$

$$u(x,0) = 0, \quad \frac{\partial u}{\partial y}(x,1) = 0 \quad (0 < x < 2);$$

$$u(0,y) = 100, \quad \frac{\partial u}{\partial x}(2,y) = 0, \quad (0 < y < 1).$$

3. (17%) Derive the D'Alembert's solution of the following wave equation for $u(x, t)$ with the given initial conditions.

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, \quad (-\infty < x < \infty, \; 0 < t < \infty);$$

$$u(x,0) = f(x), \; u_t(x,0) = g(x) \quad (-\infty < x < \infty).$$