

I

1. Solve  $y'' + 2y' + y = 2x \sin x$  (17%)

2. Solve the following differential equation: (17%)

$$(x^3 D^3 - x^2 D^2 - 7xD + 16I)y = 9x \ln x$$

3. Find a general solution of: (17%)

$$y' = \begin{bmatrix} -3 & 1 \\ 1 & -3 \end{bmatrix} y + \begin{bmatrix} -6 \\ 2 \end{bmatrix} e^{-2t}$$

Note: Laplace Transform (For your reference only, if you prefer to use it.)

f(t)	1	T	$e^{at}$	$te^{at}$	$\sin(\omega t)$	$u(t-a)$
L(f)	$1/s$	$1/s^2$	$1/(s-a)$	$1/(s-a)^2$	$\omega/(s^2+\omega^2)$	$e^{-as}/s$

$$L(f') = sL(f) - f(0)$$

$$L(f'') = s^2 L(f) - sf(0) - f'(0)$$

## Linear Algebra and Vector Calculus (Part II)

Evaluate the integral  $I = \int_C (3x^2 dx + 2yz dy + y^2 dz)$  if  $C$  has the initial point  $A: (0, 1, 2)$  and terminal point  $B: (1, -1, 7)$ . (Hint: By the Potential Theorem) (17%)

Find the inverse  $\mathbf{A}^{-1}$  of (17%)

$$\mathbf{A} = \begin{bmatrix} 2 & 0 & 1 \\ -2 & 3 & 4 \\ -5 & 5 & 6 \end{bmatrix}$$

Find the eigenvalues and eigenvectors of the matrix (17%)

$$\mathbf{A} = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

博士班資格考 工程數學 (C 卷)

1. (17%) Solve the following partial differential equation for  $u(x, t)$  first and then plot the distribution of  $u(x, t)$  vs.  $x$  at different  $t$ 's.

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad (0 < x < L, \quad 0 < t < \infty);$$

$$\text{BCs: } u(0, t) = 0, \quad u(L, t) = 100, \quad (0 < t < \infty).$$

$$\text{IC: } u(x, 0) = 0.$$

2. (a) (10%) Find the Fourier cosine and Fourier sine transforms of  $f(x) = e^{-ax}$  ( $a > 0$ ) and then (b) (7%) find the Fourier transform of  $f(x) = e^{-ax}$  if  $x > 0$  and  $f(x) = 0$  if  $x < 0$ ; here  $a > 0$ .

3. Solve the following partial differential equations for  $u(x, y)$ .

(a)  $\frac{\partial u}{\partial y} + u = e^{-xy}$  (9%)

(b)  $\frac{\partial^2 u}{\partial y^2} = u$  (8%)