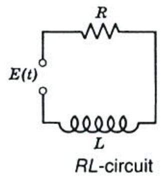
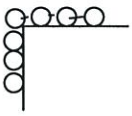
Part I

1. Write down the governing differential equations of the following problems.
   (a) For a R-L circuit with input voltage E(t). (5%)



   (b) 

   A chain of length "a" so that length "b" dangles over the side. Find the equation to check how long it will take for the chain to slide off the table? Assume the mass density is ρ. (6%)

   (c) Write down the modeling differential equations and derive the following equations of motion with constant acceleration. Where s is distance, t is time, a is acceleration, v_f is final velocity, and v_0 is initial velocity. (6%)

   $$s = v_o t + \frac{1}{2}at^2,\ v_f = v_0 + at,\ v_f^2 = v_0^2 + 2as$$

2. Solve the following differential equations:
   (a) $x^2 y''' - 3x^2 y'' + 6xy' - 6y = x^4 \ln x,\ (x > 0)$ (9%) (Hint, use method of variations of parameters, i.e. wronskian)
   (b) $xy'' - y = 0$ (8%)

3. Solve the following system of differential equations:
   (a) $y'' + 2y' + 2y = e^{-t} + 5\delta(t-2),\ y(0) = 0,\ y'(0) = 1$ (8%)
   (b) $\begin{cases} y_1'' = 4y_2 - 4e^t \\ y_2'' = 3y_1 + y_2 \end{cases}$ (9%)

   Note: You may use Laplace Transform

f(t)	1	t	e^{at}	te^{at}	$\sin(\omega t)$	u(t-a)
L (f)	1/s	$1/s^2$	1/(s-a)	$1/(s-a)^2$	$\omega/(s^2+\omega^2)$	e^{-as}/s

   $$L(f') = sL(f) - f(0)$$
   $$L(f'') = s^2 L(f) - sf(0) - f'(0)$$

Linear Algebra and Vector Calculus (Part II)

4、 Evaluate the integral $I = \int_C [(y^2 - 6xy + 6)dx + (2xy - 3x^2)dy]$ if C has the initial point A: (-1, 0) and terminal point B: (3, 4). **(17%)**

5、 Find the inverse $\mathbf{A}^{-1}$ of **(17%)**

$$\mathbf{A} = \begin{bmatrix} 2 & 0 & 1 \\ -2 & 3 & 4 \\ -5 & 5 & 6 \end{bmatrix}$$

6、 Find the eigenvalues and eigenvectors of the matrix **(17%)**

$$\mathbf{A} = \begin{bmatrix} 6 & -1 \\ 5 & 4 \end{bmatrix}$$

博士班資格考　　工程數學　　　　Part III

7、(17%) Solve the following partial differential equation for $u(x, t)$ first and then plot the distribution of $u(x, t)$ vs. x at different t's.

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad (0 < x < L, \ 0 < t < \infty);$$

BCs: $u(0,t) = 0$, $u_x(L,t) = 0$, $(0 < t < \infty)$.

IC: $u(x,0) = x$

8、(17%) Find the two basic half-range expansions (even and odd) of the following function. Sketch $f(x)$ and its two periodic extensions.

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \end{cases}$$

9. (17%) Solve the following Laplace equation with the given boundary conditions.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad (0 < x < a, \ 0 < y < b);$$

$$\frac{\partial u}{\partial y}(x,0) = 0, \quad \frac{\partial u}{\partial y}(x,b) = 0 \quad (0 < x < a);$$

$$u(0,y) = 100, \quad u(a,y) = 0, \quad (0 < y < b).$$