

1. (a) What is the meaning of an integrating factor for an ordinary differential equation (ODE) such as:  $P(x, y)dx + Q(x, y)dy = 0$
- (b) Derive the equation of the integrating factor for the ODE of (a) if it depends on  $x$  only, i.e.  $F(x)$ .
- (c) For the following linear ODE, find its integrating factor.
- $$y' + p(x)y = r(x)$$

Hint for (c): Separate the variables  $x$  and  $y$ , transform it into the format of equation in (a), then use processes of (b) to find the integrating factor.

2. Solve the following differential equations:

(a)  $y'' + 4y' + 5y = 25x^2 + 13\sin 2x$

(b)  $y''' + y' = 0$

3. Solve the following system of differential equations:

$$\begin{cases} y_1'' = 16y_2 \\ y_2'' = 16y_1 \end{cases}, y_1(0) = 2, y_1'(0) = 12, y_2(0) = 6, y_2'(0) = 4$$

Note: You may use Laplace Transform

$f(t)$	1	$T$	$e^{at}$	$te^{at}$	$\sin(\omega t)$	$u(t-a)$
$L(f)$	$1/s$	$1/s^2$	$1/(s-a)$	$1/(s-a)^2$	$\omega/(s^2+\omega^2)$	$e^{-as}/s$

$$L(f') = sL(f) - f(0)$$

$$L(f'') = s^2 L(f) - sf(0) - f'(0)$$

## Linear Algebra and Vector Calculus ( )

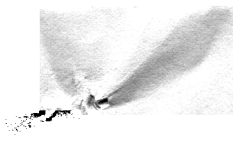
4. Find the eigenvalues and eigenvectors of the matrix (17%)

$$\mathbf{A} = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$$

5. Find the inverse  $\mathbf{A}^{-1}$  of (17%)

$$\mathbf{A} = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$$

6. Evaluate the integral  $I = \int_C [2xyz^2 dx + (x^2 z^2 + z \cos yz) dy + (2x^2 yz + y \cos yz) dz]$  if  $C$  has the initial point  $A: (0, 0, 1)$  and terminal point  $B: (1, \pi/4, 2)$ . (17%)



7. Solve the following partial differential equation for  $u(x, t)$ . (17%)

$$\frac{\partial u}{\partial t} = 4 \frac{\partial^2 u}{\partial x^2} \quad (0 < x < L, \quad 0 < t < \infty);$$

$$u(x, 0) = 100 \sin(\pi x / L), \quad (0 < x < L);$$

$$u(0, t) = 0, \quad u(L, t) = 0, \quad (0 < t < \infty).$$

8. Solve the following partial differential equation for  $u(x, t)$ . (17%)

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}, \quad (0 < x < a, \quad 0 < y < b, \quad 0 < t < \infty);$$

$$u = 0 \quad \text{on the boundary for } 0 < t < \infty,$$

$$u(x, y, 0) = f(x, y) \quad (0 < x < a, \quad 0 < y < b),$$

$$u_t(x, y, 0) = g(x, y) \quad (0 < x < a, \quad 0 < y < b).$$

9. (a) Derive the Fourier cosine and sine transforms of  $f'(x)$  in terms of the Fourier cosine and sine transforms of  $f(x)$ . (b) Use the results of (a) to find the Fourier cosine transform of  $f(x) = e^{-ax}$  ( $a > 0$ ) (17%)