96(1)博资考 工程数学

1. (a) What is the meaning of an integrating factor for an ordinary differential equation(ODE) such as: $P(x, y)dx + Q(x, y)dy = 0$

   (b) Derive the equation of the integrating factor for the ODE of (a) if it depends on x only, i.e. F(x).

   (c) For the following linear ODE, find its integrating factor.

   $y' + p(x)y = r(x)$

   Hint for (c): Separate the variables x and y, transform it into the format of equation in (a), then use processes of (b) to find the integrating factor.

2. Solve the following differential equations:

   (a) $y'' + 4y' + 5y = 25x^2 + 13\sin 2x$

   (b) $y''' + y' = 0$

3. Solve the following system of differential equations:

$$\begin{cases} y_1'' = 16y_2 \\ y_2'' = 16y_1 \end{cases}, \; y_1(0) = 2,\; y_1'(0) = 12,\; y_2(0) = 6,\; y_2'(0) = 4$$

Note: You may use Laplace Transform

f(t)	1	T	e^{at}	te^{at}	sin(ωt)	u(t-a)
L (f)	1/s	$1/s^2$	1/(s-a)	$1/(s-a)^2$	$\omega/(s^2+\omega^2)$	e^{-as}/s

$$L(f') = sL(f) - f(0)$$

$$L(f'') = s^2 L(f) - sf(0) - f'(0)$$

Linear Algebra and Vector Calculus (   )

4.Find the eigenvalues and eigenvectors of the matrix **(17%)**

$$\mathbf{A} = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$$

5.Find the inverse $\mathbf{A}^{-1}$of **(17%)**

$$\mathbf{A} = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$$

6.Evaluate the integral $I = \int_C [2xyz^2 dx + (x^2z^2 + z\cos yz)\, dy + (2x^2yz + y\cos yz) dz]$ if C has the initial point A: (0, 0, 1) and terminal point B: $(1,\ \pi/4, 2)$. **(17%)**

7. Solve the following partial differential equation for $u(x, t)$. (17%)

$$\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2} \quad (0 < x < L,\ 0 < t < \infty);$$

$$u(x,0) = 100\sin(\pi x / L), \quad (0 < x < L);$$

$$u(0,t) = 0,\ u(L,t) = 0, \quad (0 < t < \infty).$$

8. Solve the following partial differential equation for $u(x, t)$. (17%)

$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}, \quad (0 < x < a,\ 0 < y < b,\ 0 < t < \infty);$$

$u = 0$ on the boundary for $0 < t < \infty$,

$$u(x, y, 0) = f(x, y) \quad (0 < x < a,\ 0 < y < b),$$

$$u_t(x, y, 0) = g(x, y) \quad (0 < x < a,\ 0 < y < b).$$

9. (a) Derive the Fourier cosine and sine transforms of $f'(x)$ in terms of the Fourier cosine and sine transforms of $f(x)$. (b) Use the results of (a) to find the Fourier cosine transform of $f(x) = e^{-ax}$ $(a > 0)$ (17%)