機械所博士班資格考　工程數學　試題

October 2006

1. (16 %)　Solve the differential equation.

(1) (5 %)　$2xyy' = y^2 - x^2$

(2) (5 %)　$y' + y\tan x = \sin 2x$

(3) (6 %)　$y'' + 2y' + 5y = 1.25e^{0.5x} + 40\cos 4x - 55\sin 4x, \quad y(0) = 0.2,\ y'(0) = 60.1$

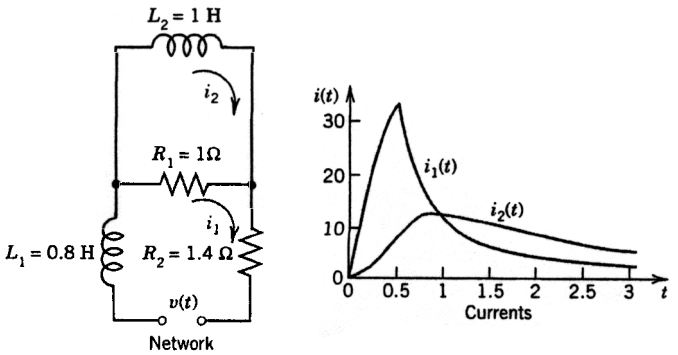
2. (17 %)　Use the power series method to solve $(x^2 - x)y'' - xy' + y = 0$

3. (17 %)　Solve the initial value problem by means of **Laplace transforms**

$$0.8i_1' + 1(i_1 - i_2) + 1.4i_1 = 100\left[1 - u\left(t - \frac{1}{2}\right)\right]$$

$$1i_2' + 1(i_2 - i_1) = 0$$

Find the currents $i_1(t)$ and $i_2(t)$ in the network with L and R measured in terms of the usually units, $v(t)$= 100 volts if $0 \le t \le 0.5$ sec and 0 thereafter, and $i(0) = 0$, $i'(0) = 0$.



Linear Algebra and Vector Calculus

4. Find the inverse $\mathbf{A}^{-1}$ of **(17%)**

$$\mathbf{A} = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$$

5. Diagonalize the matrix. (Hint: $\mathbf{D} = \mathbf{X}^{-1}\mathbf{A}\,\mathbf{X}$) **(17%)**

$$\mathbf{A} = \begin{bmatrix} 7.3 & 0.2 & -3.7 \\ -11.5 & 1.0 & 5.5 \\ 17.7 & 1.8 & -9.3 \end{bmatrix}$$

6. Evaluate the integral $I = \int_C (3x^2 dx + 2\,y\,z\,dy + y^2 dz)$ if C has the initial point A: (0, 1, 2) and terminal point B: (1, -1, 7). (Hint: By the Potential Theorem) **(17%)**

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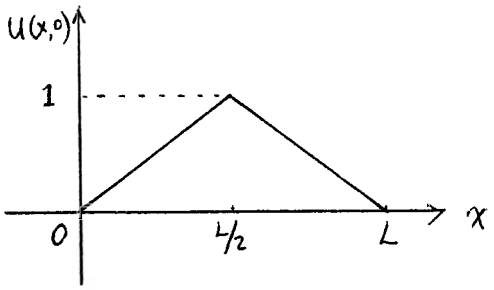
7. (17%) Solve the following partial differential equation for $u(x, t)$ and plot the distribution of $u(x, t)$ vs. x at different t's.

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \quad (0 < x < L, \ 0 < t < \infty);$$

BCs: $u_x(0,t) = 0$, $u(L,t) = 0$, $(0 < t < \infty)$.

IC:



8. (17%) Transform $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ in (x, y) into polar coordinates (r, θ) with r $\sqrt{x^2 + y^2}$ and $\theta = \arctan\frac{y}{x}$.

9. (17%) Solve the following partial differential equations for $u(x, y)$:

(a) $u_{xx} - u = 0$　(8%)

(b) $u_{xy} = -u_x$　(9%)