工

機械所博士班資格考　工程數學　試題

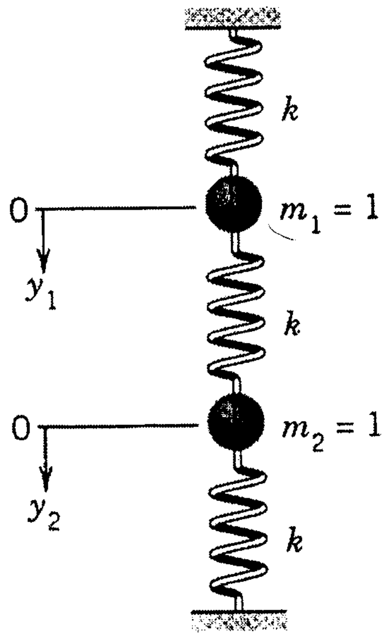
October 2005

1. (17 %) Solve the initial value problem.

$$y''+2y'+5y = 1.25e^{0.5x} + 40\cos 4x - 55\sin 4x \qquad y(0) = 0.2 \qquad y'(0) = 60.1$$

2. (17 %) Solve $(x^2 - x)y'' - xy' + y = 0$ by the Frobenius method.

3. (17 %) The mechanical system in the following figure consists of two bodies of mass 1 on three springs. k is the spring constant of each of the three springs, and y_1 and y_2 are the displacements of the bodies from their positions of static equilibrium. The masses of the springs and damping are neglected. The differential equations follow from Newton's second law for a single body. Determine the solution of this initial value problem by means of **Laplace transforms** corresponding to the initial conditions $y_1(0)=1,\ y_2(0)=1,\ y_1'(0)=\sqrt{3k},\ y_2'(0)=-\sqrt{3k}$. (17 %)



Linear Algebra and Vector Calculus

Find the eigenvalues and eigenvectors of the matrix **(17%)**

$$\mathbf{A} = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$$

Show that the integral $\int_C \mathbf{F} \bullet d\mathbf{r} = \int_C (2x\,dx + 2y\,dy + 4z\,dz)$ is independent of path in any domain in space and find its value if C has the initial point A: (0, 0, 0) and terminal point B: (2, 2, 2). **(17%)**

We know that in a body heat will flow in the direction of decreasing temperature. Physical experiments show that the rate of flow is proportional to the gradient of the temperature. This means that the velocity $\mathbf{v}$ of the heat flow in a body is of the form

$$\mathbf{v} = -K \text{ grad } U$$

where $U(\mathrm{x}, \mathrm{y}, \mathrm{z}, \mathrm{t})$ is temperature, t is time and K is called the thermal conductivity of the body; in ordinary physical circumstances K is a constant. Using this information, set up the mathematical model of heat flow, the so-called **heat equation**. (Hint: By the Divergence Theorem) **(17%)**

II

博士班資格考　　工程數學（C 卷）

1. (17%) Solve the following Laplace equation with the given boundary conditions.

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, \quad (0 < x < a,\ 0 < y < b);$$
$$u(x,0) = 0, \quad u(x,b) = f(x) \quad (0 < x < a);$$
$$u(0,y) = g(y), \quad u(a,y) = 0, \quad (0 < y < b).$$

2. (17%) Solve the following partial differential equation of $u(x, y)$ with given initial conditions by using the variable transformation: $v = x + ct$ and $z = x - ct$.

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}, \quad (-\infty < x < \infty,\ 0 < t < \infty);$$

$$u(x,0) = f(x), \quad u_t(x,0) = g(x) \quad (-\infty < x < \infty).$$

3. (17%) Derive the complex form of the Fourier integral

$$f(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(v) e^{iw(x-v)} dv dw$$

from the (real) Fourier integral

$$f(x) = \int_0^{\infty} [A(w) \cos wx + B(w) \sin wx] dw .$$