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機械所博士班資格考　工程數學　試題

April 2005

1. (16 %) Find a general solution.

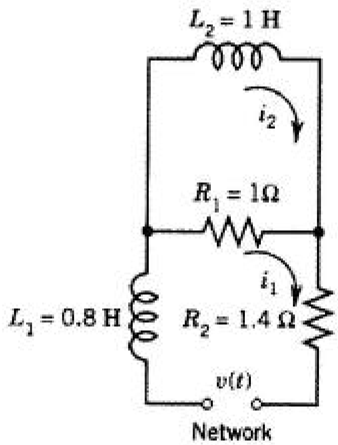
(1) $y' + \frac{1}{3}y = \frac{1}{3}(1-2x)y^4$ (8 %)

(2) $y'' - 4y' + 4y = e^{2x}/x$ (8 %)

2. (17 %) Find a basis of solutions by the Frobenius method.

$$xy'' + 3y' + 4x^3 y = 0$$

3. (17 %) By Kirchhoff's voltage law, show the model of the currents in the network in the figure. And find the currents $i_1(t)$ and $i_2(t)$ with L and R measured in terms of henry (H) and ohms (Ω), $v(t) = 100$ volts if $0 \le t \le 0.5$ sec and 0 thereafter, and $i_1(0) = 0$, $i_2(0) = 0$.



Network

Ⅱ

Linear Algebra and Vector Caculus

1. 若 $y=\mathbf{x}^{\mathrm{T}}\mathbf{A}\mathbf{x}=\mathbf{x}^{\mathrm{T}}\begin{bmatrix}3 & 1\\ 1 & 2\end{bmatrix}\mathbf{x}$，$|\mathbf{x}|=1$，(a) 請問 A 矩陣的特徵值與特徵向量為何？
   (b) 請問 y 的極大值與極小值為何？ (c) 當 **x** 等於多少時 y 會有極大值？
   (d) 請舉出一個本題在機械系統上的應用實例。

2. 若向量場 $\vec{\mathbf{F}}=(x+z)\cdot\vec{\mathbf{i}}+2y\cdot\vec{\mathbf{j}}+(x+z)\cdot\vec{\mathbf{k}}$，請計算 $\int_C \vec{\mathbf{F}}\cdot d\vec{\mathbf{r}}$ 由 A 點 (1, 1, 0) 到 B 點 (2, 3, 5) 的積分值。又，請問此積分值為何與路徑無關？

3. The flow of heat in a temperature field takes place in the direction of maximum decrease of temperature *T*. Find (a) the direction in general for the temperature field on a flat plate $T(x,y)=1/(2x^2+3y^2)$, (b) the direction at the point *P*: (2, 1). (c) 請寫出通過 P 點的等溫度線的切線方程式。

博士班資格考　　工程數學（C卷）

1. (17%) Use the concepts of linearity and superposition to break the following problem into several simpler ones, in each simple problem there is only one nonhomogeneous boundary condition.

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0,\ (0<x<a,\ 0<y<b);$$

$$u(x,0)=f(x),\ \ u_y(x,b)=0\ \ (0<x<a);$$

$$u_x(0,y)=g(y),\ u(a,y)=h(y),\ (0<y<b).$$

2. (17%) Solve the following partial differential equation for u(x, t) by using separation of variables.

$$\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2},\ (0<x<1,\ 0<t<\infty);$$

$$u(x,0)=f(x),\ u_t(x,0)=g(x)\ \ \ (0<x<1);$$

$$u(0,t)=0,\ u_x(1,t)=0,\ (0<t<\infty).$$

3. (17%) Represent the following function *f(x)* by (a) a Fourier series, (b) a Fourier cosine series, and (c) a Fourier sine series. Graph the corresponding periodic extensions of *f(x)*, respectively? (17%)

$$f(x)=\begin{cases}1 & 0<x<1\\ 0 & 1<x<2\end{cases}$$