Engineering Mathematics

PhD Candidate Qualification Examination

Institute of Mechanical Engineering, Yuan Ze University

Oct. 2002

Part I (Ordinary Differential Equation and Laplace Transform)

1.Solve the linear differential equation

$(1+x^2)(dy-dx) = 2xy\, dx$ (17%)

2. Solve the ordinary differential equation

$y'' + y = \sec x$ (17%)

3.Using the Laplace transform, solve the following problem

$F(t) = t$ if $0 < t < 2$ and $F(t) = 2$ if $2 < t$

find the $L\{F(t)\}$ (17%)

機械所博士班資格考　工程數學　試題

October 2002

(每題 17 分)

4. (1) Evaluate $\int_C \vec{F}\cdot\vec{r}'ds$, $\vec{F}=(x^2+y^2)^{-1}[-y, x, 0]$, C the circle $x^2+y^2=1$, $z=0$, oriented clockwise. Why can Stoke's theorem not be applied?

   (2) Evaluate $\iint_S \vec{F}\cdot\vec{n}dA$ by the divergence theorem. $\vec{F}=[1, 1, 1]$, $S: x^2+y^2+4z^2=4$, $z\geq 0$.

5. Prove the following formulas.

   (1) $div(f\,\vec{v}) = f\,div\,\vec{v} + \vec{v}\cdot\nabla f$

   (2) $div(f\,\nabla g) - div(g\nabla f) = f\nabla^2 g - g\nabla^2 f$

6. An elastic membrane in the x_1x_2-plane with boundary circle $x_1^2+x_2^2=1$ is stretched so that a point $P:(x_1, x_2)$ goes over into the point $Q:(y_1, y_2)$ given by

$$y=\begin{bmatrix}y_1\\y_2\end{bmatrix}=Ax=\begin{bmatrix}5&3\\3&5\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix};\text{ in components, }\begin{matrix}y_1=5x_1+3x_2\\y_2=3x_1+5x_2\end{matrix}.$$

Find the **principal directions**, that is, the directions of the position vector x of P for which the direction of the position vector y of Q is the same or exactly opposite. What shape does the boundary circle take under this deformation?

工程數學: **(Fourier Analysis and Partial Differential Equations)** Oct. 2002

7. Let us assume that *f(x)* is a periodic function of period 2π and is integrated over a period. Let us further assume that *f(x)* can be represented by a trigonometric series, **(17%)**

$$f(x) = a_0 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$$

Also, we have the so-called Euler formulas,

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx dx \qquad n = 1,2,\cdots,$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx dx \qquad n = 1,2,\cdots,$$

These numbers are called the Fourier coefficients of *f(x)*.

__Here is the question__: The periodic function *f(x)* is shown in Fig.1 and the formula is,

$$f(x) = k \quad \text{if} \quad 0 < x < \pi$$
$$f(x) = -k \quad \text{if} \quad -\pi < x < 0 \quad \text{and} \quad f(x + 2\pi) = f(x)$$

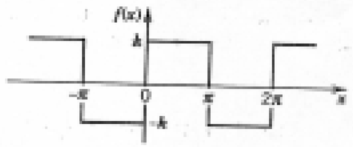

Fig. 1 The given function *f(x)* (Periodic square wave)

Please find the following questions:

(a) Find the Fourier coefficient of the periodic function *f(x)*

(b) Please briefly draw the first three partial sums of the corresponding Fourier series.

8. Please find solutions *u(x,y)* of the following partial differential equations. **(17%)**

(a) Solve the partial differential equation $u_{xy} = -u_x$.

(b) Find a solution *u(x, y)* of the partial differential equation $u_y = u$.

9. The general expression of the Fourier transform of *f(x)* is in the following: **(17%)**

$$\hat{f}(w) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-iwx} dx$$

Please explain the following question and find the Fourier transform of the following function $f(x)$

(a) What are the differences of the Fourier transform and Discrete Fourier transform (DFT) ? And, why did we need to learn them ?

(b) Please find the Fourier transform of *f(x)*

$f(x) = -1$ if $-a < x < 0$,

$f(x) = 1$ if $0 < x < a$, and

$f(x) = 0$ otherwise.