元智大學 機械工程研究所　博士班資格考試題

科目：工程數學

April 2001

1. Find a general solution.

(1) $y'+\left(2x^4-\frac{1}{x}\right)y=x^3y^2+x^5$ (6 points)

(2) $x^3y'''-3x^2y''+6xy'-6y=x^4\ln x$ (6 points)

(3) $y^{(4)}+10y''+9y=40\sinh x$ (5 points)

2. Find the displacement $w(x,t)$ of an elastic string subject to the following conditions.

(1) The string is initially at rest on the x-axis from $x=0$ to ∞ (semi-infinite string)

(2) For time $t>0$ the left end of the string is moved in a given fashion, namely,

$$w(0,t)=f(t)=\begin{cases}\sin t & \text{if } 0\le t\le 2\pi\\ 0 & \text{otherwise}\end{cases}$$

(3) Furthermore,

$$\lim_{x\to\infty} w(x,t)=0 \qquad \text{for} \quad t\ge 0$$

(The model describes a long string or rope of negligible weight with its right end fixed far out on the x-axis)

(17 points)

3. Show that the problem consisting of

$$u_t-c^2u_{xx}=Ne^{-\alpha x}$$

and the boundary conditions $u(0,t)=0,\quad u(L,t)=0$ for all t

the initial condition $u(x,0)=f(x)$

can be reduced to a problem for the homogeneous heat equation by setting

$$u(x,t)=v(x,t)+w(x)$$

and determining w so that v satisfies the homogeneous equation and the conditions $v(0,t)=v(L,t)=0,\quad v(x,0)=f(x)-w(x)$.

(17 points)

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4. Find the solution of the following set of equations using Laplace transforms and inverse Laplace transforms.

$$\frac{dx_1}{dt} = -\frac{8}{100}x_1 + \frac{2}{100}x_2 + 6 \quad \text{with} \quad x_1(0) = 0$$

$$\frac{dx_2}{dt} = \frac{8}{100}x_1 - \frac{8}{100}x_2 \quad \text{with} \quad x_2(0) = 150$$

5. Determine the response of the damped mass-spring system governed by

$$y'' + 3y' + 2y = r(t), \qquad y(0) = 0, \qquad y'(0) = 0$$

where *r(t)* is:

(a) the square wave $r(t) = u(t-1) - u(t-2)$

(b) the unit impulse at time $t = 1$, $r(t) = \delta(t-1)$

(*Hint*: The Laplace transform of the $u(t-a) = \frac{e^{-as}}{s}$)

6. The general expression of the Fourier transform of *f(x)* is in the following:

$$\hat{f}(w) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} f(x) e^{-iwx} dx$$

Please find the Fourier transforms of the following functions $f(x)$

(a) $f(x) = k$ if $0 < x < a$ and $f(x) = 0$ otherwise.

(b) $f(x) = xe^{-x^2}$

(*Hint*: The Fourier transform of the $e^{-ax^2} = \frac{1}{\sqrt{2a}} e^{-w^2/4a}$)

7. (a)Find the principal directions and corresponding factors of extension or contraction of the elastic deformation y=Ax.

   (b) Please also diagonalize the matrix A

$x = A^{-1} Y$

$$A^{-1} = \begin{pmatrix} 0.4 & -0.1 \\ -0.2 & 0.3 \end{pmatrix}$$

$D = A$

8. Find the region beneath $z=4x^2+9y^2$ and above the rectangle with vertices (0,0), (3,0), (3,2), and (0,2)

9.Find the length of the following curve from (a,0,0) to (a,0,2 π c)

$r(t) = a\cos t\,\vec{i} + a\sin t\,\vec{j} + ct\,\vec{k}$