1. The stream function for a particular flow is given as (10 %)

$$\psi(x,y) = x^2 - y^2$$

Is this flow irrotational ? If so, calculate the velocity potential.

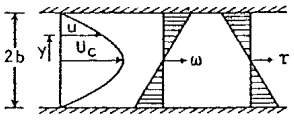
2. Find the radial and tangential velocity for the potential function $\phi = r^{\frac{1}{2}}\cos(\frac{\theta}{2})$ and the stream function. Sketch the flow. (15 %)

3. Determine the circulation of A around the triangle which has vertices at the origin (0,4,0) and (4,0,0). The vector function A is given by

$$A = 4x^2\vec{i} + 2yz\vec{j} - (4y^2 + z^2)\vec{k} \qquad (10\%)$$

4. A viscous, incompressible fluid flows between two plates. The flow is laminar and two-dimensional, the velocity profile is parabolic : (20 %)

$$u = U_c(1 - y^2/b^2)$$



1999 機械系博士班資格考流力試題 (Part 2)

1. (30%) The lift force F on a missile is a function of its length L, velocity V, diameter D, angle of attack α, and the density ρ, viscosity μ, and speed of sound a of the air. Rewrite this relation in dimensionless form.

2. (30%) A constant-thickness film of viscous liquid flows in steady laminar motion down a plate inclined at angle θ, as in Figure 1. Solve for the velocity distribution u(y) in the fluid. What is the proper boundary condition at the free surface? What will the volume flux be? (You may make any plausible assumptions.)

Figure 1

3. (a) (10%) What is the most general form of a plane polar coordinate incompressible flow which satisfies continuity and is purely circulatory; that is $v_\theta = v_\theta(r, \theta, t)$ and $v_r = 0$?
   (b) (10%) Repeat problem (a) but find the most general flow pattern which is purely radial, $v_r = v_r(r, \theta, t)$ and $v_\theta = 0$.
   (c) (20%) What are the special cases of these two fields which also satisfy the Navier-stokes equations? Can you interpret the two results physically?
   (Hint: In cylindrical coordinates,