考試科目	方式	
工程數學	Closed Book, 不可使用計算機, 共 9 題採計 6 題	Part I

Ordinary Differential Equations (Part I)

Using the method of undetermined coefficients to solve the differential equation (17%)

$$y'' + 4y = 8x^2$$

Find a solution of the following equation (17%)

$$y'' - 4y' + 4y = 0 \quad \text{with} \quad y(0) = 3 \quad, \quad \left.\frac{dy}{dx}\right|_{x=0} = 1$$

Using the method of Laplace Transformation to solve the initial value problem of $y(t)$ (17%)

$$y'' + 4y' + 3y = e^t \quad \text{with} \quad y(0) = 0 \quad, \quad \left.\frac{dy}{dt}\right|_{t=0} = 2$$

1. $A = \begin{bmatrix} 6 & -3 \\ 4 & -1 \end{bmatrix}$, (a) Find the eigenvalues and eigenvectors of A; (b) Find A^{100}; (c) Solve the following system of differential equations by using the result of (a). (6%, 5%, 6%)。

$$X' = \begin{bmatrix} 6 & -3 \\ 4 & -1 \end{bmatrix} X \quad \text{where} \quad X = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

2. If $f(x, y, z) = 50\,xy + x - z^2$, then (a) what is the value of $\dfrac{df}{ds}$ at the point (1, 0, -2) and in the direction (1, 0, 2)? (b) What is the maximum value of the derivative $\dfrac{df}{ds}$ at the same point (1, 0, -2) and in which direction does the maximum derivative occur? (9%, 8%)

3. Evaluate $\iint\limits_{s} (\bar{F} \cdot \bar{n}) dA$ when $\bar{F} = [x^2, y^2, xy]$ and S is the portion of the plane x + 2y + 3z = 1 in the first octant. (17%)

1. Solve $\dfrac{\partial u}{\partial x} = 4\dfrac{\partial u}{\partial y}$, $u(0,y) = 8e^{-3y}$. (17%)

2. Expand $f(x) = x, 0 < x < 2$ in a half range (a) sine series, (b) cosine series. (17%)

3. Find the Fourier integral representation of the function. (17%)

$$f(x) = \begin{cases} 1 & if \ |x| < 1 \\ 0 & if \ |x| > 1 \end{cases}$$