1011 機械系博士班資格考試題目

考試科目	方式	
工程數學	Closed Book, 不可使用計算機, 共 9 題採計 6 題	Part I

1. **Using the method of variation of parameters to solve the differential equation (17%)**

$$y'' + y = \sec x$$

2. **Find a solution of the following equation (17%)**

$$y'' + y' - 2y = 0 \quad \text{with} \quad y(0) = 4 \ , \ \left.\frac{dy}{dx}\right|_{x=0} = -5$$

3. **Using the method of Laplace Transformation to solve the initial value problem of $y(t)$ (17%)**

$$y'' + 2y' + y = e^{-t} \quad \text{with} \quad y(0) = -1 \ , \ \left.\frac{dy}{dt}\right|_{t=0} = 1$$

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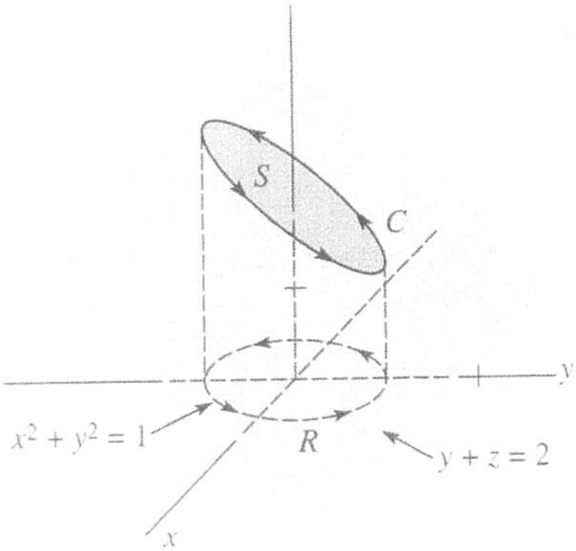
1. (a) A = $\begin{pmatrix} 4 & 2 \\ 3 & 3 \end{pmatrix}$, diagonalize A. (b) Solve the following system of differential equations by using the result of (a). (8%, 9%)。

$$X' = \begin{pmatrix} 4 & 2 \\ 3 & 3 \end{pmatrix} X \quad \text{where} \quad X = \begin{pmatrix} x_1(t) \\ x_2(t) \end{pmatrix}$$

2. What is the divergence theorem (i.e., Gauss theorem)? Verify the divergence theorem for $\bar{F} = [x, y, z]$, and D is the sphere of $x^2 + y^2 + z^2 = 9$. (5%, 12%)

3. Using Stokes's theorem, evaluate $\oint_C zdx + xdy + ydz$ where C is the trace of the cylinder $x^2 + y^2 = 1$ in the plane $y + z = 2$. Orient C counterclockwise as viewed from above. (17%)



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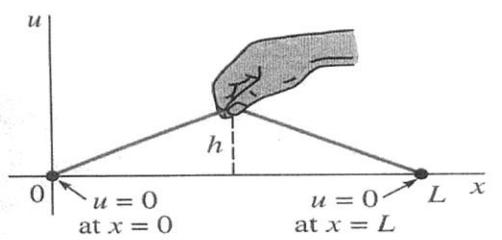
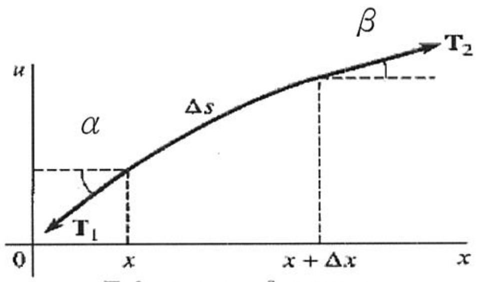
1. Solve $u_x + 3u_y = 0$ (17%)

2. For an elastic string of length L, fastened at its ends on the x-axis at x=0 and x=L. The string is displaced with the initial displacement f(x) as shown. Then it is released from test to vibrate in the x-y plane. Following the step by step instructions below, find the model for this kind of wave equations in PDE. Where $\rho =$ mass/per unit length, $c^2 = \frac{T}{\rho}$ (17%)

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$





Enlargement of segment

(a) List the force equilibrium in horizontal x-direction.

(b) List the force equilibrium in vertical direction by Newton's 2nd law. i.e. $\sum F = ma$

$$T_2 \sin\beta - \_\_\_\_\_\_ = \text{ma} = \rho\, \Delta x \frac{\partial^2 u}{\partial}$$

(c) Divide those equations as above in (a) by (b), you can obtain

$$\Rightarrow \frac{T_2 \sin\beta}{T_2 \cos\beta} - \frac{???}{T_1\, ???} = \tan\beta - ??? = \frac{\rho\, \Delta x}{T} \frac{\partial^2 u}{\partial\, ?}$$

(d) The slope at x and $x + \Delta x$, $\_\_\_\_\_ = \left(\frac{\partial u}{\partial x}\right)\Big|_x$ and $\_\_\_\_\_ = \left(\frac{\partial u}{\partial x}\right)\Big|_{x+\Delta x}$ substitute into (c) for terms on the right. You can get the partial differential equation.

3. Expand the following function by Fourier series. (17%) $f(x) = \begin{cases} x, & 0 \le x \le \frac{L}{2} \\ L - x, & \frac{1}{2} < x \le L \end{cases}$