1002 機械系博士班資格考試題目

考試科目	方式	
工程數學	Closed Book,不可使用計算機, 共 9 題採計 6 題	Part I

1. **Using the method of undetermined coefficients to solve the differential equation (17%)**

$$y'' - 3y' + 2y = e^x$$

2. **Find a solution of the following equation (17%)**

$$y'' + y' - 2y = 0 \quad \text{with} \quad y(0) = 4 \quad , \quad \left.\frac{dy}{dx}\right|_{x=0} = -5$$

3. **Using the method of Laplace Transformation to solve the initial value problem of $y(t)$ (17%)**

$$y'' - y = t \quad \text{with} \quad y(0) = 1 \quad , \quad \left.\frac{dy}{dt}\right|_{t=0} = 1$$

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1. (a) Determine the inverse A^{-1} by the method of Gauss-Jordan method. (10%)

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$$

(b) Prove $(AB)^{-1} = B^{-1} A^{-1}$ for any nonsingular matrices A and B. (7%)

2. (a) Show that the line integral $\int_C \bar{F} \cdot d\bar{r}$ is path independent in any domain in space, where $\bar{F} = 2x\bar{i} + 2y\bar{j} + 4z\bar{k}$. (9%)

(b) Evaluate the line integral of (a) by direct integration along C: $\bar{r}(t) = [t, t, t]$, $0 \le t \le 2$. (8%)

3. $\mathbf{A} = \begin{bmatrix} -5 & 2 \\ 2 & -2 \end{bmatrix}$

(a) Find the eigenvalues and eigenvectors of A. (7%)

(b) Solve the following system of ODEs by using the results of (a). (10%)

$$\bar{y}'' = A\bar{y} \quad \text{where} \quad \bar{y} = \begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix}$$

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1. Solve $\frac{\partial^2 u}{\partial x^2} + 4\frac{\partial^2 u}{\partial y^2} = 0$ (17%)

2. Fourier integral representation of a function. (17%)
   (a) What is Fourier integral? When to use it? (5%)
   (b) Solve for the Fourier integral of $f(x) = \begin{cases} 1, & \text{if } 0 < x < a \\ 0, & \text{if } x > a \end{cases}$ (12%)

3. Find the steady state oscillations of the following forced oscillations under a non-sinusoidal periodic driving force. Following the step by step instructions below. (17%)

   $y'' + \omega^2 y = r(t)$
   with $r(t)$ as given

   $$r(t) = \begin{cases} t + \pi & \text{if } -\pi < t < 0 \\ -t + \pi & \text{if } 0 < t < \pi \end{cases} \qquad r(t + 2\pi) = r(t), \ |\omega| \neq 0, 1, 3, 5, \cdots$$

   (a) Sketch $r(t)$? (3%)
   (b) Represent $r(t)$ by a Fourier series? (5%)
   (c) Solve the problem by taking the corresponding solution format following the results of (b) just as those in solving general second order differential equations. (9%)