1001 機械系博士班資格考試題目

考試科目	方式	
工程數學	Closed Book,不可使用計算機, 共 9 題採計 6 題	Part I

Ordinary Differential Equations (Part I)

Using the method of undetermined coefficients to solve the differential equation (17%)

$$y'' + 4y = 8x^2$$

Find a solution of the following equation (17%)

$$y'' - y = 0 \quad \text{with} \quad y(0) = 4 \quad , \quad \left.\frac{dy}{dx}\right|_{x=0} = -2$$

Using the method of Laplace Transformation to solve the initial value problem of $y(t)$ (17%)

$$y'' + 2y' + y = e^{-t} \quad \text{with} \quad y(0) = -1 \quad , \quad \left.\frac{dy}{dt}\right|_{t=0} = 1$$

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1. A is a diagonalizable square matrix, and D is its corresponding diagonalized matrix.

   (a) Prove: $D^m = X^{-1} A^m X$, where X is the matrix with A's eigenvectors as column vectors. (7%)

   (b) If $A = \begin{pmatrix} 4 & 2 \\ 3 & 3 \end{pmatrix}$, find A^{100}. (10%)

2. $\vec{F} = (y^2 + 3y)\vec{i} + (2xy + 3x)\vec{j}$ and C: $y = x^2$ from (0, 0) to (1, 1).

   (a) Evaluate $\int_C \vec{F} \cdot d\vec{R}$ by direct line integral. (7%)

   (b) Determine whether $\vec{F}$ is conservative in the entire x-y plane. If it is, find a potential function and then evaluate the above line integral with the potential function. (10%)

3. Evaluate $\iint_s (\vec{F} \cdot \vec{n}) dA$ when $\vec{F} = [x^2, y^2, xy]$ and S is the portion of the plane x + 2y + 3z = 1 in the first octant ($\vec{n}$ is the outer unit normal vector on S). (17%)

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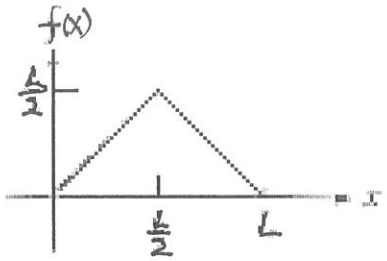
1. Expand the following function as a Fourier series. (17%)

$$f(x)=\begin{cases} x, & 0 \le x \le \dfrac{L}{2} \\ L-x, & \dfrac{L}{2} < x \le L \end{cases}$$



2. Find the Fourier transform of *f(x)*. (17%) $f(x)=\begin{cases} k, & -1 < x < 1 \\ 0, & \text{otherwise} \end{cases}$

3. To find the temperature u(x, t) in a rod of length L, if the initial temperature is f(x) throughout and if the end x=0 is kept at temperature zero and the end x=L is insulated. The governing equation is listed as following:

$$\frac{\partial u}{\partial t}=k\frac{\partial^2 u}{\partial x^2} \quad \text{for} \quad 0<x<L,\ t>0$$

(a) List the boundary conditions for x=0, and x=L (3%)

(b) List the initial condition (2%)

(c) Use separation of variable to solve the equation. (12%)